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EQUATIONS FOR BISTATIC DOPPLER SHIFT AND RATE OF CHANGE OF DOPPLER SHIFT OF DARK SATELLITE OBSERVATIONS

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CONTENTS

ABSTRACT

Equations are given for the doppler shift and rate of change of doppler shift for the bistatic case where an orbiting, nontransmitting earth satellite is illuminated by a transmitter, and the reflected energy is received at different locations on the surface of the earth. These equations have been programmed for computation by the NAREC computer for any satellite for which the orbital elements are known. The results for a number of satellites have been computed, using transmitting and receiving sites of the Space Surveillance System. Plots of various relationships between doppler shift, rate of change of doppler shift, satellite height, earth-center angle between the receiver and the satellite, and zenith angle from receiver to satellite are shown for a typical satellite, 1958 Alpha, Explorer I.

PROBLEM STATUS

This is an interim report on one phase of the problem: work is continuing.

AUTHORIZATION

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EQUATIONS FOR BISTATIC DOPPLER SHIFT AND RATE OF CHANGE OF DOPPLER SHIFT OF DARK SATELLITE OBSERVATIONS

INTRODUCTION

In order to separate reflected satellite signals from those caused by aircraft, meteor trails, lightning discharges, and direct transmitter feedthrough, the addition of doppler shift and rate of change of doppler shift data could be of considerable aid. Equations for the doppler shift f_d and rate of change of doppler shift $\dot{f}_d$ will now be developed for the bistatic case where an orbiting satellite is illuminated by a transmitter at one location on the surface of the earth and the reflected energy is received at a second location on the surface of the earth. In particular, the transmitter and receiver sites lie along a great circle which also contains several other transmitter and receiver sites. This complex of transmitters and receivers is known as the Space Surveillance System.*

The antenna configurations at each site result in a fan-shaped beam pattern with its wide dimension in the plane of the great circle. The beam patterns overlap such that whenever a satellite crosses the great circle plane it must pass through one or more beam patterns. This results in a "fence" of beam patterns, and for this reason the great circle plane is referred to as the fence plane. At each receiver, data is recorded and analyzed to determine time of passage of the satellite through the fence, the zenith angle of the satellite from the receiver at the time of the observation, and identity of the satellite.

If more than one receiving station simultaneously observes the same satellite passage, its height may be determined by triangulation.

The coordinate system used in the derivation of the equations is defined in Fig. 1. The earth is oriented such that the X axis is along the longitude of Greenwich ($0°$), the Y axis is along $90°$W longitude, and the Z axis is along the north polar axis. Thus, the XY plane corresponds to the earth's equatorial plane. Longitudes are measured positive west of Greenwich, and latitudes are measured positive towards the Z axis. The coordinate axes are fixed with respect to the earth and must therefore rotate with the earth.

Figure 2 shows the orientation of the fence plane and its great circle intersection with the surface of the earth. The locations of a receiver R and transmitter T along the great circle are also shown.

The geometry of a satellite observation in the plane of the fence is shown in Fig. 3. The separation of transmitter and receiver is exaggerated for clarity.

PROCEDURE

The NAREC computer was programmed to compute the doppler shift f_d and the rate of change of doppler shift $\dot{f}_d$ for any satellite for which the orbital elements are given. The fence plane may be any plane for which the equation is given, and the receiver and

*Proc. of the IRE 48(No. 4):663-669, "The Navy Space Surveillance System," R. L. Easton and J. J. Fleming.

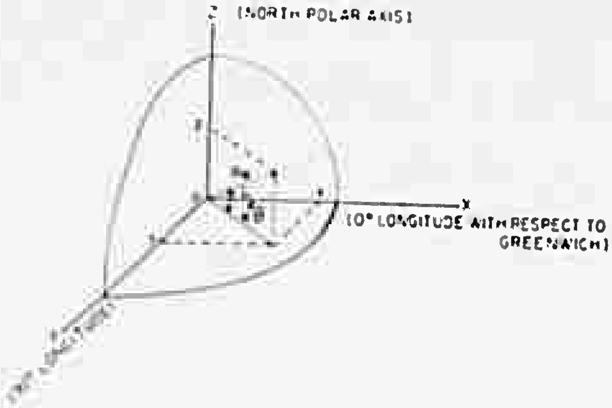


Fig. 1 - The coordinate system

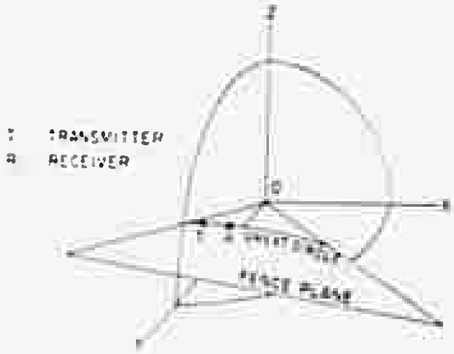


Fig. 2 - Locations of transmitter and receiver along the fence great circle

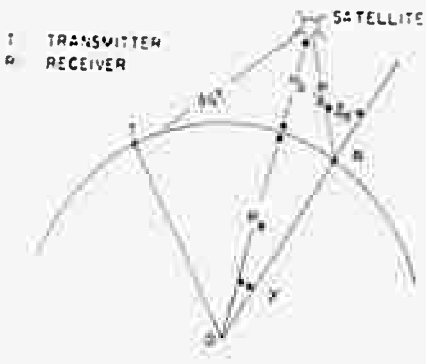


Fig. 3 - Geometry of satellite observation in plane of the fence

transmitter may be located anywhere along the fence great circle. For all the answers computed to date, the fence plane is that of the Space Surveillance System. The receiver is located at either Ft. Stewart, Georgia, or Silver Lake, Mississippi, and the transmitter is located at Jordon Lake, Alabama. The subdivision of the region above the transmitter and receiver is illustrated in Fig. 4, using the data from the satellite 1958 Alpha as an example. Again, the separation between the receiver and transmitter is exaggerated for clarity.

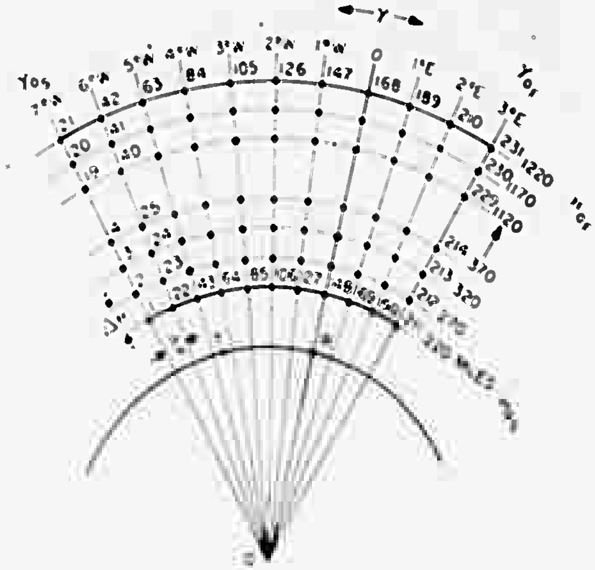


Fig. 4 - Subdivision of the region above the transmitter and receiver

The first computation was for the case where the satellite is located at point 1 at the time it crosses the fence plane. At point 1, the starting height H_{o_S} is 220 statute miles and the starting earth-center angle γ_{o_S} is 7°W. At the completion of the computation of f_{d_1} and $\dot{f}_{d_1}$ at point 1, an increment of ΔH = 50 miles is added to H_{o_S} and f_{d_2} and $\dot{f}_{d_2}$ for point 2 are computed. Another increment of ΔH is then added and the process repeated until a finishing height of H_{o_F} = 1220 miles is reached and $f_{d_{21}}$ and $\dot{f}_{d_{21}}$ are computed. At this time, the height is returned to H_{o_S}, an increment of $\Delta\gamma = 1^\circ$ is added to γ_{o_S}, and $f_{d_{22}}$ and $\dot{f}_{d_{22}}$ are computed. In this manner, f_d and $\dot{f}_d$ are computed for each of the points 1 through 231.

In the usual application of the program to the computation of f_d and $\dot{f}_d$ for actual satellites, the value of H_{o_S} is chosen as the height of perigee and H_{o_F} is chosen as the height of apogee. For all computations made to date, γ_{o_S} has been chosen as 7°W of the receiver and $\gamma_{o_F} = 3^\circ$E, although any value of γ_{o_S} and γ_{o_F} may be used. Increments of ΔH and $\Delta\gamma$ are chosen to obtain the desired subdivision of the region of interest. In the case illustrated, there are 21 increments of height and 11 increments of earth-center angle, giving a 231-point coverage of the region.

For each point at which a computation is performed, four cases must be considered, depending upon whether the fence crossing is in a north-to-south or south-to-north direction, and upon whether the satellite is approaching apogee or perigee in its orbit:

Case 1. North-south, approaching apogee (NSAA);

Case 2. North-south, approaching perigee (NSAP);

Case 3. South-north, approaching apogee (SNAA);

Case 4. South-north, approaching perigee (SNAP).

The program is written to compute answers for Case 1 first, then to return to point 1 and compute answers for Case 2. In a similar manner, answers for Cases 3 and 4 are then computed. If predictions are given for the satellite of interest, the appropriate case will be known. Otherwise, there are four sets of f_d and $\dot{f}_d$ answers for each point.

After the computer finishes computing the answers for Case 4, the computer stops and a new data tape is read into the computer with the elements of the next satellite for which answers are desired.

COMPUTER INPUTS AND OUTPUTS

Data tapes have been inserted and answers obtained for the following lists of satellites:

A. With the receiver located at Ft. Stewart and the transmitter at Jordon Lake:

1. 1958 Alpha	Explorer I
2. 1958 Beta 2	Vanguard I
3. 1959 Alpha 1	Vanguard II
4. 1959 Epsilon 2	Discoverer V Capsule
5. 1959 Eta	Vanguard III
6. 1959 Iota 1	Explorer VII
7. 1960 Beta 2	Tiros I

8. Circular Orbit with H = 100 miles: inclination = 90°.

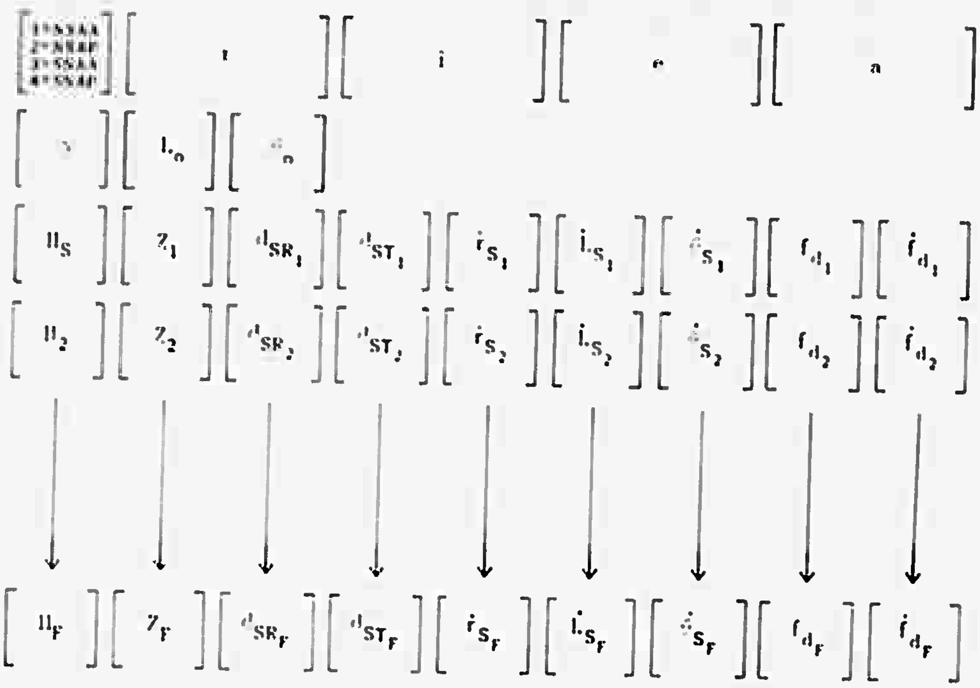
B. With the receiver located at Silver Lake and the transmitter at Jordon Lake:

1. 1958 Alpha	Explorer I
2. 1959 Alpha 1	Vanguard II
3. 1959 Epsilon 2	Discoverer V Capsule
4. 1959 Iota 1	Explorer VII
5. 1960 Beta 2	Tiros I
6. 1960 Gamma 1	Transit IB, Second Stage
7. 1960 Epsilon 1	Sputnik IV
8. 1960 Zeta 1	Midas II
9. 1960 Eta 3	Transit IIA, Second Stage
10. 1960 Iota 1	Echo I

Figure 5 is the first page of the printout of the results for 1958 Alpha, with receiver at Ft. Stewart and transmitter at Jordon Lake. The answers are divided into several groups, two of which are shown. The first group includes all the answers for each of the 21 height increments for γ = 7°W. The second group includes the answers for the 21 height increments for γ = 6°W. The succeeding pages of answers (not shown) give answers for each of the other increments of earth-center angle. The positions of the decimal points in the answers are shown in the first group, and are in the same positions in all groups. The identity of the answers in the printout may be determined by referring to Fig. 6.

1	t = 00000.000000		t = 33.210000000		t = .10847839000		t = 04687.839000	
γ = -7.0	t_o = 33.381	r_o = 099.407						
B = 00220.	7 = -69.657	d_{SR} = 054.56	d_{SI} = 0815.1	$\dot{i}_s$ = 0.0690	$\dot{i}_s$ = -.038514 x 10^{-2}	$\dot{i}_s$ = -.129253 x 10^{-2}	t_p = 4767.1	$\dot{i}_p$ = -008.84
00270	-65172	05702	08334	02434	-037611	-126038	44485	-01092
00320	-61083	05980	08562	03250	-036739	-122945	41634	-01223
00370	-57377	06286	08803	03800	-035897	-119959	38913	-01308
00420	-54029	06615	09065	04197	-035084	-117074	36331	-01356
00470	-51000	06964	09347	04485	-034298	-114286	33900	-01373
00520	-48287	07332	09646	04691	-033538	-111590	31632	-01366
00570	-45630	07714	09962	04827	-032803	-108984	29531	-01340
00620	-43000	08108	10292	04904	-032093	-106462	27596	-01301
00670	-41500	08514	10635	04929	-031405	-104021	25824	-01251
00720	-39773	08930	10991	04904	-030739	-101659	24212	-01196
00770	-38112	09354	11357	04833	-030094	-099370	22754	-01137
00820	-36597	09785	11733	04716	-029469	-097153	21445	-01075
00870	-35211	10222	12118	04553	-028863	-095005	20279	-01014
00920	-33930	10665	12512	04342	-028276	-092922	19255	-00954
00970	-32770	11112	12912	04077	-027707	-090902	18371	-00895
01020	-31602	11564	13320	03751	-027154	-088943	17631	-00839
01070	-30685	12020	13733	03348	-026618	-087042	17044	-00786
01120	-29771	12479	14152	02840	-026098	-085196	16637	-00736
01170	-29014	12941	14576	02158	-025593	-083405	16478	-00689
01220	-28115	13406	15005	00997	-025103	-081665	16881	-00646
1	00000000000		33210000000		10847839000		04687839000	
-60	33357	098210						
00220	-65716	04815	07472	00690	-041944	-127749	46534	-01172
00270	-60825	05085	07674	02434	-040960	-124579	43036	-01413
00320	-56457	05387	07902	03250	-040010	-121520	39934	-01550
00370	-52577	05718	08155	03800	-039094	-118566	37014	-01627
00420	-49138	06071	08430	04197	-038206	-115712	34280	-01657
00470	-46089	06444	08725	04485	-037352	-112953	31742	-01651
00520	-43381	06834	09038	04691	-036525	-110289	29404	-01618
00570	-40972	07237	09367	04827	-035724	-107711	27264	-01566
00620	-38821	07651	09711	04904	-034897	-105217	25316	-01502
00670	-36894	08075	10068	04929	-034201	-102803	23552	-01430
00720	-35162	08507	10437	04904	-033478	-100466	21962	-01354
00770	-33599	08947	10816	04833	-032773	-098203	20539	-01276
00820	-32184	09392	11205	04716	-032093	-096010	19273	-01198
00870	-30898	09843	11602	04553	-031433	-093885	18158	-01123
00920	-29726	10298	12007	04342	-030794	-091825	17189	-01050
00970	-28653	10757	12419	04077	-030174	[illegible]	16364	-00981
01020	-27668	11220	12837	03751	-029572	[illegible]	15684	-00916
01070	-26762	11686	13260	03348	-028989	[illegible]	15160	-00854
01120	-25926	12154	13689	02840	-028422	[illegible]	14818	-00798
01170	-25152	12625	14123	02158	-027872	[illegible]	14725	-00746
01220	-24434	13098	14561	00997	-027338	[illegible]	15201	-00700

Fig. 5 - First page of printout of answers



where t = time, in seconds, for which all equations are evaluated (t = 0 for all computations made to date);

i = inclination, in degrees, of the orbital plane;

e = eccentricity of the orbit;

a = semimajor axis, in miles, of the orbit;

θ = earth-center angle between receiver and satellite, °W → -, °E → -;

L_o = latitude of satellite at first height in the group;

λ_o = longitude of satellite at first height in the group;

Z_n = zenith angle, in degrees, from receiver to satellite;

d_{SR} = distance, in miles, between receiver and satellite;

d_{ST} = distance, in miles, between transmitter and satellite;

$\dot{r}_S$ = rate of change of radius vector, in miles per second (also equal to rate of change of height);

$\dot{L}_S$ = rate of change of latitude, in degrees per second;

$\dot{\lambda}_S$ = rate of change of longitude, in degrees per second;

f_d = doppler shift, in cycles per second;

$\dot{f}_d$ = rate of change of doppler shift, in cps per second.

Fig. 6 - Code of the answers on the computer printout (Fig. 5)

Certain information, such as orbital elements, starting and finishing heights, and starting and finishing earth-center angles, is required for the data tape input to the computer for each satellite for which answers are desired; the information required is the following:

e, eccentricity of the orbit;

a, semimajor axis of the orbit, in statute miles;

i, inclination of the orbital plane, in degrees;

t, time for which answers are to be computed;

ϵ, selected tolerance within which successive iterations in the approximation to the solution to Kepler's equation must fall;

γ_{os}, starting earth-center angle, in degrees;

γ_{of}, finishing earth-center angle, in degrees;

$\Delta\gamma$, selected increments of earth-center angle, in degrees;

H_{os}, starting height, in statute miles;

H_{of}, finishing height, in statute miles;

ΔH, selected increments of height, in statute miles.

In addition, certain other data are included on a second input tape which contains quantities which are either always constant or remain constant during several changes of the data tapes. The information on the Constants Tape includes the following:

L_R, latitude of the receiver, in degrees;

λ_R, longitude of the receiver, in degrees;

L_T, latitude of the transmitter, in degrees;

λ_T, longitude of the transmitter, in degrees;

A, coefficient of the x term in the equation of the fence plane;

B, coefficient of the y term in the equation of the fence plane;

C, coefficient of the z term in the equation of the fence plane;

R_e, radius of the earth, in statute miles;

ω_e, angular rotation rate of the earth, in radians per mean solar second;

π, the constant.

RESULTS

Some of the results for 1958 Alpha, Case 1 (north-to-south, approaching apogee), are shown graphically in Figs. 7, 8, and 9. Figure 7 is a plot of rate of change of doppler shift vs doppler shift for a family of curves of constant height and a family of curves of constant earth-center angle. The data for Case 2 (north-to-south, approaching perigee), if plotted on the same graph, would result in a closed curve for each curve in the family of constant earth-center angle. For clarity, only the data for Case 1 are shown.

Figure 8 is a plot of rate of change of doppler shift vs zenith angle for a family of curves of constant earth-center angle. As with Fig. 7, the data for Case 2 could also be included on Fig. 8 to form a family of closed curves.

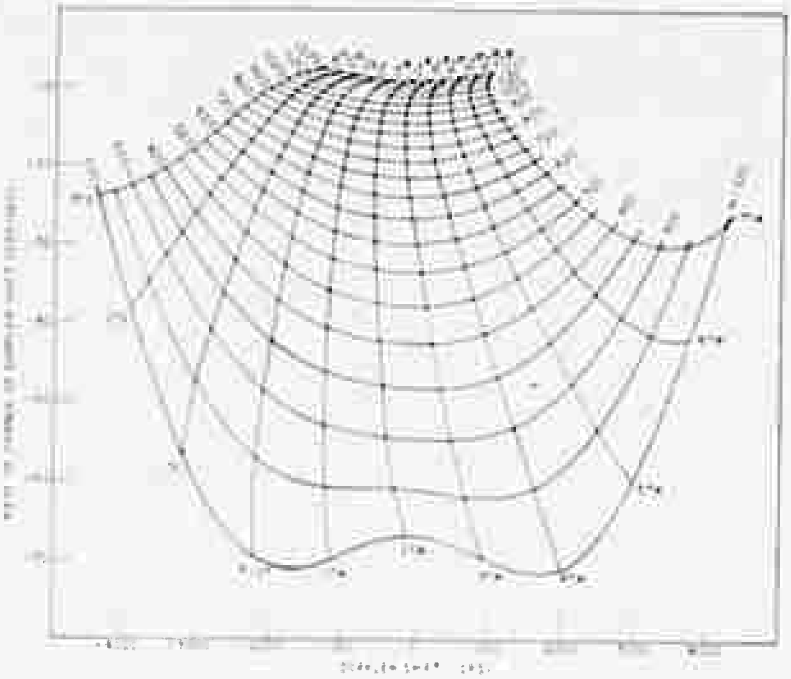


Fig. 7 - Rate of change of doppler shift vs doppler shift for curves of constant height and constant earth-center angle

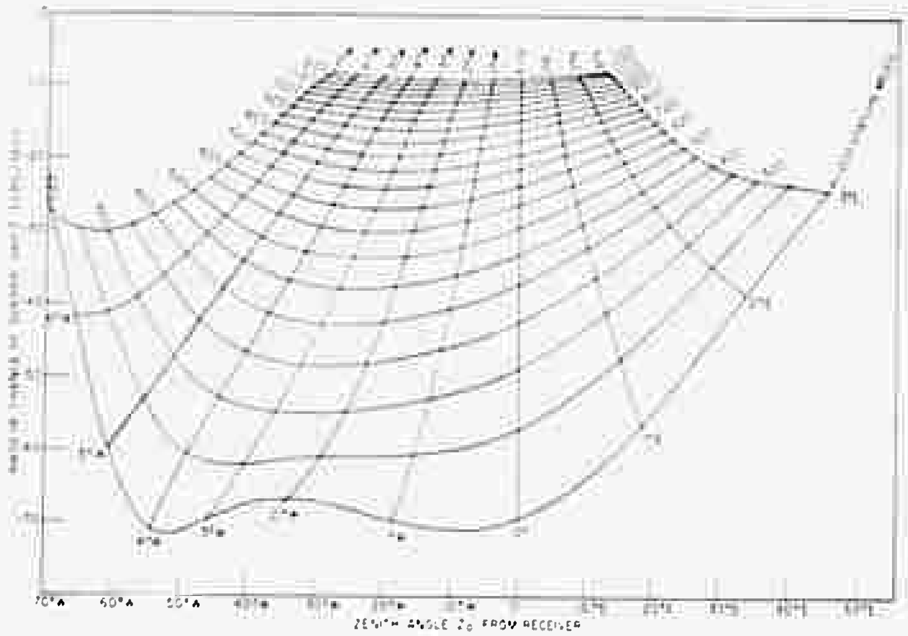


Fig. 8 - Rate of change of doppler shift vs zenith angle for curves of constant height and constant earth-center angle

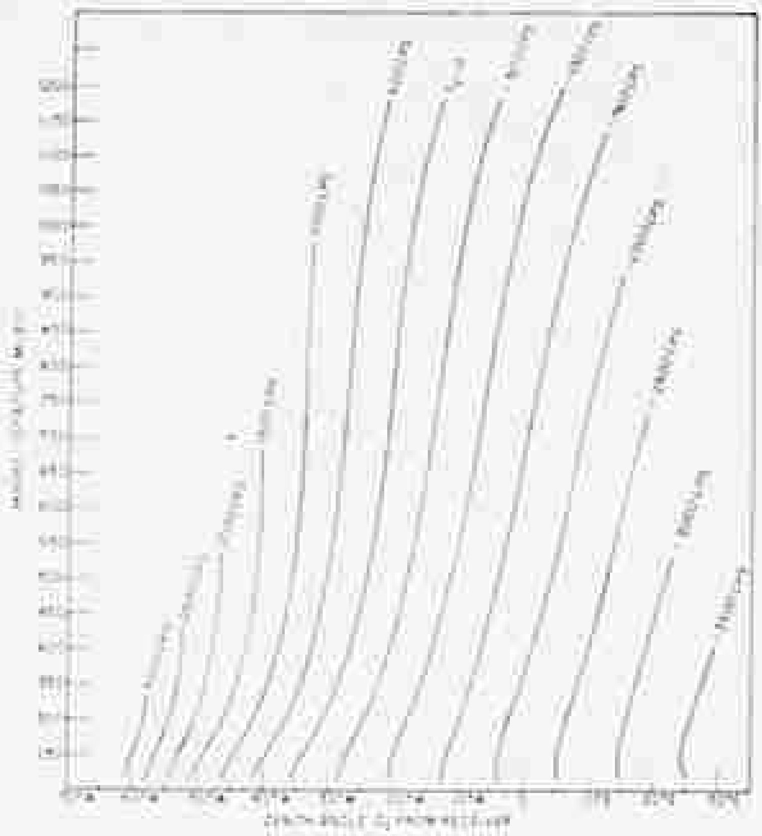


Fig. 9 - Height vs zenith angle for curves of constant doppler shift

By means of Figs. 7 and 8, data were obtained to plot Fig. 9, a plot of height vs zenith angle for a family of curves of constant f_d.

The computer printouts of answers, one page of which is shown in Fig. 5, have been compiled into two convenient booklets (an original and a carbon copy) for each of the eighteen computer runs. The answer tapes are on file from which any number of printouts may be made using any Flexowriter with a carriage 105 or more spaces wide.

EQUATIONS

The equations which were used in the computer program will now be given. Symbols used in the equations and in the text are summarized in Appendix A. The doppler shift and rate of change of doppler shift are:

doppler shift: $$f_d = -\frac{f_T}{c}(\dot{d}_{SR} + \dot{d}_{ST})$$

rate of change of doppler shift: $$\dot{f}_d = -\frac{f_T}{c}(\ddot{d}_{SR} + \ddot{d}_{ST})$$

where

f_T = transmitted frequency;

c = propagation constant;

d_{SR} = distance between satellite and receiver;

d_{ST} = distance between satellite and transmitter.

Time variable expressions for d_{SR} and d_{ST} are required so that they may be differentiated to obtain their first and second time derivatives:

$$d_{SR}(t) = \left\{[x_S(t) - x_R]^2 + [y_S(t) - y_R]^2 + [z_S(t) - z_R]^2\right\}^{1/2}$$

and

$$d_{ST}(t) = \left\{[x_S(t) - x_T]^2 + [y_S(t) - y_T]^2 + [z_S(t) - z_T]^2\right\}^{1/2}$$

where $[x_S(t), y_S(t), z_S(t)]$ are the time variable rectangle coordinates of satellite position; $[x_R, y_R, z_R]$ are the rectangular coordinates of the receiver; and $[x_T, y_T, z_T]$ are the rectangular coordinates of the transmitter. The rectangular coordinates of satellite position are obtained from the spherical coordinates by the following transformations:

$$x_S(t) = r_S(t)\,\lambda_S(t)$$

$$y_S(t) = r_S(t)\,\mu_S(t)$$

$$z_S(t) = r_S(t)\,\nu_S(t)$$

where $r_S(t)$ is the time-dependant radius vector of the satellite, and $[\lambda_S(t), \mu_S(t), \nu_S(t)]$ are the time-dependant direction cosines of satellite position. The direction cosines of satellite position are defined by the following three simultaneous equations:

$$\lambda_S^2(t) + \mu_S^2(t) + \nu_S^2(t) = 1$$

$$a_o'(t)\,\lambda_S(t) + b_o'(t)\,\mu_S(t) + c_o'(t)\,\nu_S(t) = 0$$

$$\lambda_o'(t)\,\lambda_S(t) + \mu_o'(t)\,\mu_S(t) + \nu_o'(t)\,\nu_S(t) = \cos\theta(t)$$

where $[a_o'(t), b_o'(t), c_o'(t)]$ are the coefficients of the satellite plane; $[\lambda_o'(t), \mu_o'(t), \nu_o'(t)]$ are the direction cosines of the satellite after the earth's rotation but before orbital motion; and $\theta(t)$ is the earth-center angle through which the satellite moves in its orbital plane. The coefficients in the equation of the satellite plane are found as follows:

$$a_o'(t) = \frac{A'(t)}{(A^2 + B^2 + C^2)^{1/2}}$$

$$b_o'(t) = \frac{B'(t)}{(A^2 + B^2 + C^2)^{1/2}}$$

$$c_o'(t) = \frac{C'(t)}{(A^2 + B^2 + C^2)^{1/2}}$$

where $[A'(t), B'(t), C'(t)]$ are the coefficients in the equation of the satellite plane before normalization. These coefficients are:

$$A'(t) = -B \sin \omega_p t + A \cos \omega_p t$$

$$B'(t) = A \sin \omega_p t + B \cos \omega_p t$$

$$C'(t) = C$$

where (A, B, C) are the coefficients in the equation of the satellite plane at $t = 0$:

$\omega_p = \omega_e + \omega_\Omega$,

ω_e = angular rotation rate of earth,

ω_Ω = precession of the plane of the satellite,

t = elapsed time since fence crossing;

$A = \nu_o$,

$B = -K\nu_o$, and

$C = K\mu_o - \lambda_o$

where $[\lambda_o, \mu_o, \nu_o]$ are the direction cosines of the satellite at $t = 0$, and K is defined by the following quadratic equation:

$$(\mu_o^2 \sin^2 i - \nu_o^2 \cos^2 i) K^2 - (2\lambda_o \mu_o \sin i) K + (\lambda_o^2 \sin^2 i - \nu_o^2 \cos^2 i) = 0.$$

Here, i is the inclination of the satellite plane. The earth-center angle α through which the satellite moves in its orbital plane during time t is given by the equation

$$\alpha(t) = v_S(t) - v_o + \omega_\omega t$$

where $v_S(t)$ = time variable expression for true anomaly; v_o = true anomaly at $t = 0$; and ω_ω = angular rotation rate of perigee. Now,

$$v_S(t) = \arccos \left(\frac{\cos E_S(t) - e}{1 - e \cos E_S(t)} \right)$$

where $E_S(t)$ = time variable expression for eccentric anomaly, and e = eccentricity of the orbit. To obtain $E_S(t)$, the transcendental equation known as Kepler's equation must be solved:

$$E_S(t) = M_S(t) + e \sin E_S(t)$$

where

$M_S(t) = n(t - \tau)$ = time variable expression for mean anomaly,

$n = \frac{2\pi}{P}$ = mean angular velocity of satellite in its orbit,

P = anomalistic period of the orbit,

$$\tau = -\frac{M_o}{n} = \text{time of last perigee},$$

$$M_o = E_o - e \sin E_o = \text{mean anomaly at } t = 0,$$

$$E_o = \frac{a - r_o}{ae} = \text{eccentric anomaly at } t = 0,$$

$$a = \text{semimajor axis of the orbit, and}$$

$$r_o = \text{radius vector of the satellite at } t = 0.$$

The direction cosines of the satellite, considering the earth's rotation, but before orbital motion, are defined as follows:

$$\lambda_o'(t) = \cos L_o'(t) \cos \psi_o'(t)$$

$$\mu_o'(t) = \cos L_o'(t) \sin \psi_o'(t)$$

$$\nu_o'(t) = \sin L_o'(t)$$

where

$$L_o'(t) = L_o,$$

$$\psi_o'(t) = \psi_o + \omega_p t,$$

$$\omega_p = \omega_e - \omega_\Omega,$$

$$\omega_e = \text{angular rotation rate of earth, and}$$

$$\omega_\Omega = \text{precession of the plane of the satellite.}$$

Also,

$$L_o = \arcsin \nu_o, \text{ and}$$

$$\psi_o = \arcsin \left(\frac{\mu_o}{[\mu_o^2 + \lambda_o^2]^{1/2}} \right)$$

where $[\lambda_o, \mu_o, \nu_o]$ are the direction cosines of the satellite at $t = 0$. The direction cosines $[\lambda_o, \mu_o, \nu_o]$ of the satellite at $t = 0$ are defined by the following three simultaneous equations:

$$\lambda_o^2 + \mu_o^2 + \nu_o^2 = 1$$

$$a_F \lambda_o + b_F \mu_o + c_F \nu_o = 0$$

$$\lambda_R \lambda_o + \mu_R \mu_o + \nu_R \nu_o = \cos \gamma_o$$

where $[a_F, b_F, c_F]$ are the coefficients of the fence plane equation; $[\lambda_R, \mu_R, \nu_R]$ are the direction cosines of the receiver; and γ_o is the earth-center angle between receiver and satellite at $t = 0$. Given the satellite height H_o and the zenith angle Z_o from the receiver, the earth-center angle γ_o may be found by means of the equation

$$\gamma_o = Z_o - \arcsin\left(\frac{R_e \sin Z_o}{R_e + H_o}\right)$$

where R_e is the earth's radius. The time variable expression for satellite height is:

$$H_S(t) = r_S(t) - R_e$$

where $r_S(t) = a[1 - e \cos E_S(t)]$ is the radius vector of the satellite, a and e are as defined before, and $E_S(t)$ = time variable expression for eccentric anomaly (previously defined by Kepler's equation).

CONCLUSIONS

As implied by the plot shown in Fig. 7, doppler shift and rate of change of doppler shift are sufficient to define the position of a satellite for which semimajor axis, inclination, and eccentricity are known. However, due to the relatively short duration of time that a satellite is normally in the beam of the Space Surveillance System, it is difficult to obtain a sufficiently accurate measure of the rate of change of doppler shift to make use of a plot of this kind. For other systems, where several seconds or more are available, rate of change of doppler shift could be measured with sufficient accuracy to define satellite position.

Since the zenith angles of observations are determined from other data recorded by the Space Surveillance System, the addition of doppler information to this data makes possible the determination of the height of a satellite with known orbital elements from an observation by a single station, rather than requiring triangulation by coincident observations by two or more stations. A plot such as that shown in Fig. 9 would be useful for this purpose. A program modification could be made such that the height corresponding to an observed zenith angle and doppler shift could be computed directly, so that it would not be necessary to refer to a plot. Otherwise, four plots (one for each of the four cases) must be prepared for each known satellite. This task could be simplified by having the NAREC computer answer-tape punched in the proper format so that the plots could be made on the automatic plotter.

Experimental doppler data suitable to verify the results have not been available. Various program modifications, such as indicated below, may be incorporated later should such data indicate the necessity.

1. Consider height above mean earth's radius of the receiver and transmitter.

2. Correct earth's radius for oblateness of the earth when computing satellite heights.

3. Consider decay of the semimajor axis.

If the coordinates of both the transmitter and the receiver are equal, implying that both are at the same location, the bistatic case reduces to the monostatic case, for which the equations given in this report hold equally well.

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* * *

APPENDIX A

DEFINITION OF SYMBOLS

a	Semimajor axis (given as one of the orbital elements)
$\{a_f, b_f, c_f\}$	Coefficients of the fence plane equation (given)
$\{A, B, C\}$	Coefficients of the satellite plane equation at $t = 0$
$\{A'(t), B'(t), C'(t)\}$	Coefficients of the satellite plane equation considering the earth's rotation and precession of the node
$\{a_o'(t), b_o'(t), c_o'(t)\}$	Normalized coefficients of the satellite plane equation considering the earth's rotation and precession of the node
c	Propagation constant (given)
$d_{SR}(t)$	Distance between satellite and receiver
$d_{ST}(t)$	Distance between satellite and transmitter
e	Eccentricity (given as one of the orbital elements)
E_o	Eccentric anomaly at $t = 0$
$E_S(t)$	Eccentric anomaly at any time t
$f_d(t)$	Doppler shift
$\dot{f}_d(t)$	Rate of change of doppler shift
f_T	Transmitted frequency
ΔH	Selected increments of height
H_o	Height of the satellite at $t = 0$
H_{o_S}	Starting height of the subdivision of the region above the transmitter and the receiver
H_{o_F}	Finishing height of the subdivision of the region above the transmitter and the receiver
$H_S(t)$	Height of the satellite at any time t
i	Inclination of the satellite plane
K	Constant, defined by the quadratic equation:

$$(\mu_o^2 \sin^2 i - \nu_o^2 \cos^2 i)\, K^2 - (2\lambda_o \mu_o \sin i)\, K + (\lambda_o^2 \sin^2 i - \nu_o^2 \cos^2 i) = 0$$

L_o	Latitude of the satellite at $t = 0$
$L_o'(t)$	Latitude of the satellite, considering only the earth's rotation and precession of the node
$L_S(t)$	Latitude of the satellite at any time t
M_o	Mean anomaly at $t = 0$
$M_S(t)$	Mean anomaly at any time t

n	Mean angular velocity of the satellite in its orbit
P	Anomalistic period (given as one of the orbital elements)
r_o	Radius vector of the satellite at $t = 0$
$r_S(t)$	Radius vector of the satellite at any time t
R_e	Radius of the earth (given)
v_o	True anomaly at $t = 0$
$v_S(t)$	True anomaly at any time t
$[x_S(t), y_S(t), z_S(t)]$	Rectangular coordinates of the satellite at any time t
$[x_R, y_R, z_R]$	Rectangular coordinates of the receiver
$[x_T, y_T, z_T]$	Rectangular coordinates of the transmitter
Z_o	Zenith angle of a satellite as observed from the receiver (given)
$\theta(t)$	Angle in the plane of the satellite through which the satellite moves during time t
$\Delta\gamma$	Selected increments of earth-center angle
γ_o	Earth-center angle between the satellite and the receiver at $t = 0$
γ_{o_S}	Starting earth-center angle of the subdivision of the region above the transmitter and the receiver
γ_{o_F}	Finishing earth-center angle of the subdivision of the region above the transmitter and the receiver
ϵ	Tolerance within which successive iterations in the approximation to the solution of Kepler's equation must fall
ϕ_o	Longitude of the satellite at $t = 0$
$\phi_o'(t)$	Longitude of the satellite at any time t considering only the earth's rotation and precession of the node
$\phi_S(t)$	Longitude of the satellite at any time t
$[\lambda_R, \mu_R, \nu_R]$	Direction cosines of the receiver
$[\lambda_o, \mu_o, \nu_o]$	Direction cosines of the satellite at $t = 0$
$[\lambda_o'(t), \mu_o'(t), \nu_o'(t)]$	Direction cosines of the satellite at any time t considering the earth's rotation and precession of the node
$[\lambda_S(t), \mu_S(t), \nu_S(t)]$	Direction cosines of the satellite at any time t
τ	Time satellite was last at perigee before $t = 0$
ω_e	Angular rotation rate of the earth (given)
ω_p	Apparent angular rotation rate of the satellite plane
ω_Ω	Precession of the node (given)
ω_ω	Rotation of perigee (given)

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